Statistics

Fall 2022

Lecture 25

Given $.724 < P < .866$

1) Find the point-estimate $\hat{p}$

$\hat{p} = \frac{.866 + .724}{2} = \boxed{.795}$

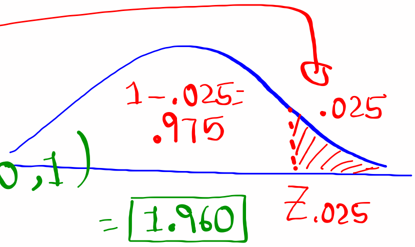
2) Find $\hat{q}$

$\hat{q} = 1 - \hat{p} = 1 - .795 = \boxed{.205}$

3) Find the margin of error, round to whole%.

$E = \frac{.866 - .724}{2} = \boxed{.071} = 7.1\%$ $\boxed{7\%}$

Find $Z_{.025}$



$Z_{.025} = \text{invNorm}(.975, 0, 1)$

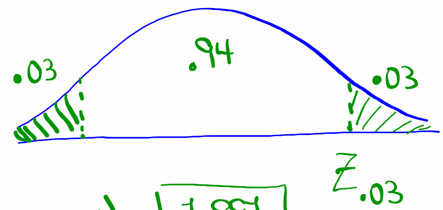
$= \boxed{1.960}$

Find $Z_{\alpha/2}$ for 94% Conf. level.

Middle Area .94

$1 - .94 = .06$

$.06 \div 2 = .03$



$Z_{.03} = \text{invNorm}(.97, 0, 1) = \boxed{1.881}$

Given $n = 180$, $x = 81$, C-level: .99

Find Conf. interval for pop. proportion.

1-PropZInt

$x = 81$

$n = 180$

C-level: .99

$\boxed{.354 < P < .546}$

$\hat{p} = \frac{.546 + .354}{2} = \boxed{.45}$

$E = \frac{.546 - .354}{2} = \boxed{.096}$

Given $n = 225$, $\hat{p} = .32$, C-level: Not given

Find conf. interval for pop. proportion.

$x = n\hat{p}$

if decimal → Round-up

$= 225(.32)$

$= 72$

1-PropZInt

$x = 72$

$n = 225$

C-level: .95

$\boxed{.259 < P < .381}$

$\hat{p} = \frac{.381 + .259}{2} = \boxed{.32}$ $E = \frac{.381 - .259}{2} = \boxed{.061}$

Among 755 randomly selected students, 14% of them were drinkers.

Find 98% conf. interval for the prop. of all students that are drinkers.

$n = 755$

$\hat{p} = .14$

$x = n\hat{p}$

if decimal → Round-up

$= 755(.14) = 105.7$ $\boxed{x = 106}$

$\hat{p} = \dfrac{.170 + .111}{2} = \boxed{.141}$

$E = \dfrac{.170 - .111}{2} = \boxed{.0295} \approx 3\%$

1-PropZInt

$x = 106$

$n = 755$

C-level: .98

$\boxed{.111 < p < .170}$

we are 98% confident that between 11% and 17% of all students are drinkers.

Suppose we wish to construct $\boxed{96\%}$ conf. interval for the prop. of all and margin of error not exceed 4%, find minimum sample size needed if

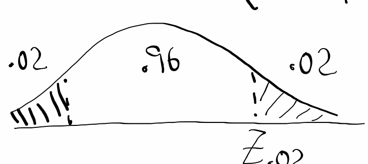
a) $\hat{p} = .15$

$n = \hat{p} \cdot \hat{q} \left(\dfrac{Z_{\alpha/2}}{E} \right)^2$

If decimal → Round-up

$= (.15)(.85) \left(\dfrac{2.054}{.04} \right)^2 = 336.195$

$\boxed{n \approx 337}$



$Z_{.02} = \text{invNorm}(.98, 0, 1) = 2.054$

b) $\hat{p}$ & $\hat{q}$ are both unknown.

$n = .25 \left(\dfrac{Z_{\alpha/2}}{E} \right)^2$

$= .25 \left(\dfrac{2.054}{.04} \right)^2$

$= 659.206$

$\boxed{n \approx 660}$

SG 22

Confidence Interval for population Mean μ:

$$\bar{x} - E < \mu < \bar{x} + E$$

Sample Mean "Point-Estimate"

Margin of error

Case I: σ Known

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

STAT | TESTS | Z Interval

inpt: STATS

$$\bar{x} = \frac{\quad + \quad}{2}$$

$$E = \frac{\quad - \quad}{2}$$

Given: $n = 35$, $\boxed{\bar{x} = 82}$, $\sigma = 12$, C-level: .9

Find Confidence Interval for pop. mean.

Since σ is Known

$78.664 < \mu < 85.336$

$$\boxed{79 < \mu < 85}$$

$$\bar{x} = \frac{85 + 79}{2} = \boxed{82}$$

$$E = \frac{85 - 79}{2} = \boxed{3}$$

Z Interval

Inpt: STATS

$\sigma = 12$

$\bar{x} = 82$

$n = 35$

C-level: .9

Calculate

30 randomly selected nurses had a mean monthly salary of \$6200.

$n=30$
$\bar{x}=6200$

It is known that standard deviation of monthly salaries of all nurses is \$500.

$\sigma=500$

Find 99% conf. interval for the mean monthly salaries of all nurses.

C-level: .99

$5964.9 < \mu < 6435.1$

Since σ is known
=> Use Z Interval

$\boxed{5965 < \mu < 6435}$

inpt: $\boxed{\text{Stats}}$

$\bar{x} = \frac{\quad + \quad}{2}$

$\sigma : 500$

$\bar{x} = 6200$

$n = 30$

C-level: .99

$E = \frac{6435 - 5965}{2} = \boxed{235}$

Confidence Interval for population Mean μ:

$$\bar{x} - E < \mu < \bar{x} + E$$

Sample Mean "Point-Estimate"

Margin of error

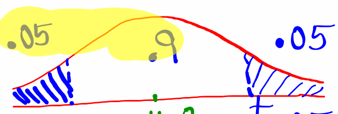
Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ df $= n-1$
STAT TESTS Z Interval inpt: STATS	T Interval

$\bar{x} = \frac{\quad + \quad}{2}$

$E = \frac{\quad - \quad}{2}$

Given: $n=15$, $\bar{x}=88$, $S=10$, C-level: .9

Find $t_{\alpha/2}$



$t_{.05} = \text{invT}(.95, 14)$
$= \boxed{1.761}$

σ unknown
$df = n-1 = 14$

Find conf. interval for pop. mean.
$< \mu <$

σ unknown
⇒ T Interval

Inpt: STATS
$\bar{x}=88$
$S=10$
$n=15$
C-level: .9

$83.452 < \mu < 92.548$

$\boxed{83 < \mu < 93}$

$\bar{x} = \frac{93+83}{2} = \boxed{88}$

$E = \frac{93-83}{2} = \boxed{5}$

I randomly selected 20 students, their mean age was 29.8 Yrs with standard deviation of 9.5 Yrs.

$n=20$, $\bar{x}=29.8$, $S=9.5$

Find Conf. interval for the mean age of all Students.

No C-level
Use .95

$\boxed{25.4 < \mu < 34.2}$

σ unknown ⇒ T Interval

$\bar{x} = \frac{34.2+25.4}{2} = \boxed{29.8}$

$E = \frac{34.2-25.4}{2} = \boxed{4.4}$

I randomly Selected 10 exams, here are the Scores:

75	83	95	68
100	90	70	88
58	98		

Find

1) $\bar{x} = 83$

2) $S = 14$

Round to whole #

3) Find 98% Conf. interval for the mean of all exams.

$\boxed{71 < \mu < 95}$

σ uknown $\Rightarrow$ T Interval

$E = \frac{95 - 71}{2} = \boxed{12}$

How to determine minimum Sample Size when constructing Confidence Interval for Pop. mean:

$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $\Rightarrow$ Solve for n (with Some algebra)

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

when decimal $\Rightarrow$ Round-up

If σ is unknown $\Rightarrow$ use S in place of σ.

$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E}\right)^2$$

Find min. Sample Size needed for Constructing 88% Conf. interval for pop. mean with margin of error not to exceed 5 assuming $\sigma = 30$.

C-level: .88

$Z_{.06}$

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

$$= \left(\frac{1.555 \cdot 30}{5} \right)^2$$

$Z_{.06} = \text{invNorm}(.94, 0, 1)$

$= 1.555$

$= 87.0489$

$\boxed{n \approx 88}$

Given: $S = 20$, $E = 4$, C-level: .99

Find min. Sample Size needed to construct conf. interval for pop. mean.

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

Since σ is unknown, we use S instead.

$$n = \left(\frac{2.576 \cdot 20}{4} \right)^2$$

$Z_{.005} = \text{invNorm}(.995, 0, 1)$

$= \boxed{2.576}$

$n = 165.8944$

$n = 166$

Redo with $E = 10$

$$n = \left(\frac{2.576 \cdot 20}{10} \right)^2$$

$\boxed{\text{SG } 23 \checkmark\checkmark\checkmark}$

$n = 27$